A Logic for Hybrid Rules

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RuleML Workshop 2006

Outline

Motivation – Hybrid Knowledge Bases

Combine **rules with negation as failure** with **classical theories**:

- **Hyrid KB** approaches rely on (variants of) the Answer Set Semantics. [Rosati,2005/2005b/2006, Heymans, et al. 2006]
- Defined for **syntactically limited** programs/FOL theories
- All give a **modular definition** of models by projection+reduct.

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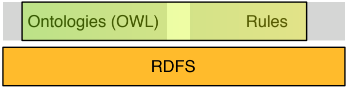

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- Does this provide us with notions of equivalence commonly used (strong equivalence, uniform equivalence, etc.)?

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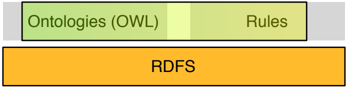

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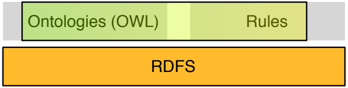

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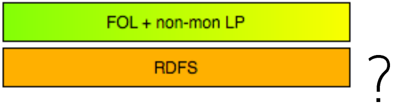

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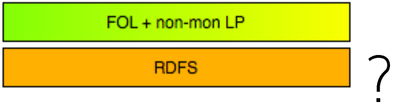

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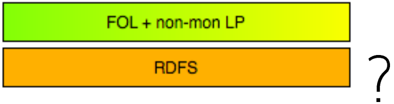

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Hybrid Knowledge Bases – Generalized Definition

$\mathcal{K} = (\mathcal{T}, \mathcal{P})$ *hybrid knowledge base*:

- classical first-order theory $\mathcal{T}$ over function-free language $\mathcal{L}_\mathcal{T} = \langle C, P_\mathcal{T} \rangle$
- a logic program $\mathcal{P}$ over function-free language $\mathcal{L} = \langle C, P_\mathcal{T} \cup P_\mathcal{P} \rangle$, i.e. a set of rules:

$$a_1 \vee a_2 \vee \ldots \vee a_k \vee \neg a_{k+1} \vee \ldots \vee \neg a_l \leftarrow b_1, \ldots, b_m, \neg b_{m+1}, \ldots, \neg b_n$$

  where $P_\mathcal{T} \cap P_\mathcal{P} = \emptyset$

Note:

- $\mathcal{T}$ and $\mathcal{P}$ talk about the same constants, and
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Hybrid Knowledge Bases – Projection:

Overall idea for a nonmonotonic semantics: "evaluate" $\mathcal{P}$ wrt a classical model of the theory and then compute stable models.

Let $\mathcal{P}$ be a *ground* program an $\mathcal{I} = \langle U, I \rangle$ an $\mathcal{L}$-structure, with $U = (D, \sigma)$.

$\Pi(\mathcal{P}, \mathcal{I})$, the projection of $\mathcal{P}$ wrt $\mathcal{I}$, obtained by

1. deleting each rule with head literal $p(t)$ (or $\neg p(t)$) over $At_D(C, P_T)$ such that $p(\sigma(t)) \in I$ (or $p(\sigma(t)) \notin I$)
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Example – a small Hybrid KB:

Let $\mathcal{K} = (\mathcal{T}, \mathcal{P})$ with

$\mathcal{T}$: Each foaf:Person is a foaf:Agent:

$\forall x.PERSON(x) \rightarrow AGENT(x)$
$AGENT(David)$

$\mathcal{P}$: Some nonmonotonic rule on top

$PERSON(x) \leftarrow pcmember(x, LPNMR), AGENT(x), \neg machine(x)$
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Is David a PERSON?

Recall: $\neg$ is "negation as failure" here!

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Classical models of $\mathcal{T}$:

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$gr_U(\mathcal{P})$

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$PERSON(David)$ in all NM-models, i.e. $\mathcal{K} \models_{NM} PERSON(David)$

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Equilibrium Logic

Back to our question for a non-classical logic which covers this. . .

- *Equilibrium logic* (Pearce, 1997) generalizes stable model semantics and answer set semantics for logic programs to arbitrary propositional theories.
- It is a nonmonotonic extension of the logic of Here-and-there (with strong negation).
- Model theory based on Kripke semantics for intuitionistic logic
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Quantified Here-and-there Logic

- $\mathbf{QHT^s}$ is complete for linear Kripke frames with two worlds "here" and "there" with a "static" domain over both worlds: $h \leq t$.
- here-and-there structures: $\mathcal{M} = \langle (D, \sigma), I_h, I_t \rangle$
- I_h, I_t are first-order-interpretations over D such that $I_h \subseteq I_t$.

The models are extended to all formulas via the rules known in intuitionistic logic, notions of validity and logical consequence relation are the ones for (intuitionistic) Kripke semantics.

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Quantified Here-and-there Logic

For $w \in \{h, t\}$:

- $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$.
- $\mathcal{M}, w \models \varphi \vee \psi$ iff $\mathcal{M}, w \models \varphi$ or $\mathcal{M}, w \models \psi$.
- $\mathcal{M}, t \models \varphi \rightarrow \psi$ iff $\mathcal{M}, t \not\models \varphi$ or $\mathcal{M}, t \models \psi$.
- $\mathcal{M}, h \models \varphi \rightarrow \psi$ iff $\mathcal{M}, t \models \varphi \rightarrow \psi$ and $\mathcal{M}, h \not\models \varphi$ or $\mathcal{M}, h \models \psi$.
- $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, t \not\models \varphi$.
- $\mathcal{M}, t \models \forall x \varphi(x)$ iff $\mathcal{M}, t \models \varphi(d)$ for all $d \in D$.
- $\mathcal{M}, h \models \forall x \varphi(x)$ iff $\mathcal{M}, t \models \forall x \varphi(x)$ and $\mathcal{M}, h \models \varphi(d)$ for all $d \in D$.
- $\mathcal{M}, w \models \exists x \varphi(x)$ iff $\mathcal{M}, w \models \varphi(d)$ for some $d \in D$.

Quantified Equilibrium Logic (QEL)

- We write QHT^s-structures more briefly as ordered pairs of atoms $\langle H, T\rangle$, with $H \subseteq T$.
- An QHT^s-Structure $\langle H, T\rangle$ is said to be total if $H = T$
- Order relation: $\langle H, T\rangle \trianglelefteq \langle H', T'\rangle$ if $T = T'$ and $H \subseteq H'$
- $\langle H, T\rangle$ is an equilibrium model of Π if is
  - (i) $\langle H, T\rangle$minimal under $\trianglelefteq$, and
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QEL is determined by the equilibrium models of a theory.

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Quantified Equilibrium Logic and Answer Set Semantics

- Equilibrium Logic generalizes Answer Set semantics for arbitrary formulae (including disjunctive and nested programs)
- Any rule

  $a_1 \vee a_2 \vee \ldots \vee a_k \vee \neg a_{k+1} \vee \ldots \vee \neg a_l \leftarrow b_1, \ldots, b_m, \neg b_{m+1}, \ldots, \neg b_n$

  is just treated as (universally closed) formula in QEL:

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Embedding Hybrid Knowledge Bases

Q: Does the correspondence extend to hybrid KBs? Yes!

Idea: define embedding based on the observation that adding LEM makes intuitionistic logic classical!

Given a hybrid KB $\mathcal{K} = (\mathcal{T}, \mathcal{P})$ we call $\mathcal{T} \cup st(\mathcal{T}) \cup \mathcal{P}$ the stable closure of $\mathcal{K}$, where $st(\mathcal{T}) = \{\forall x(p(x) \vee \neg p(x)) : p \in \mathcal{L}_\mathcal{T}\}$.

Wake up! Main theorem of the paper!!! ;-)

Theorem

Let $\mathcal{K} = (\mathcal{T}, \mathcal{P})$ be a hybrid knowledge base. Let $\mathcal{M} = \langle U, T, T \rangle$ be a total here-and-there model of the stable closure of $\mathcal{K}$. Then $\mathcal{M}$ is an equilibrium model if and only if it is an NM-model of $\mathcal{K}$.

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Example - *stable closure* of $\mathcal{K}$:

$st(\mathcal{K}) = \mathcal{T} \cup st(\mathcal{T}) \cup \mathcal{P}$

$\forall x.PERSON(x) \rightarrow AGENT(x)$
$AGENT(David)$
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There IS a classical model of this theory
$\mathcal{M} = \{\neg PERSON(David), machine(David), \ldots\}$

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- Quantified Equilibrium Logic provides a powerful and intuitive tool as a "carrier" logic for Hybrid KBs
- Embedding is simple: add LEM for classical predicates.
- Why this works is not so surprising: QHT^s based on intuitionistic logic, adding LEM enforces totalization of HT models on the respective predicates, i.e. make them "classical".
- No reducts/grounding involved, this gives us:
  - a semantics for nested logic programs. Well-investigated for propositional LPs, first-order case needs more investigation, respective results on QEL relatively new.
  - results on strong equivalence for Equilibrium Logic carry over: HT-model equivalence amounts to strong equivalence! Important notion for nonmonotonic logic programs, program optimization, meaning preserving transformations of hybrid KB's, etc.

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Future Work:

- Paper covers only equality-free FOL embedding (results already there in [Pearce&Valverde, 2006])
- Investigation on related (IJCAI) approaches:
  - Logic of minimal knowledge and negation as failure (MKNF) [Motik & Rosati, 2007]
  - First-Order Autoepistemic Logic [de Bruijn et al., 2007]
  - Circumscription [Ferraris,Lee,Lifschitz, 2007]

Get the paper at: `http://polleres.net/publications.html`

Variants discussed in the paper:

	UNA	variables	disj.	neg. $\mathcal{L}_\mathcal{T}$ atoms
r-hybrid [Rosati,2005]	yes	$\mathcal{L}_\mathcal{P}$-safe	pos.	no
r^+-hybrid [Rosati,2005b]	no	$\mathcal{L}_\mathcal{P}$-safe	pos.	no
r_w-hybrid [Rosati, 2006]	yes	weak $\mathcal{L}_\mathcal{P}$-safe	pos.	no
g-hybrid [Heymans, et al. 2006]	no	guarded	neg.*	yes

* g-hybrid allows negation in the head but at most one positive head atom

Table: Different variants of hybrid KBs